

## Fourteenth International Olympiad, 1972

### 1972/1.

Prove that from a set of ten distinct two-digit numbers (in the decimal system), it is possible to select two disjoint subsets whose members have the same sum.

### 1972/2.

Prove that if  $n \geq 4$ , every quadrilateral that can be inscribed in a circle can be dissected into  $n$  quadrilaterals each of which is inscribable in a circle.

### 1972/3.

Let  $m$  and  $n$  be arbitrary non-negative integers. Prove that

$$\frac{(2m)!(2n)!}{m!n!(m+n)!}$$

is an integer. ( $0! = 1$ .)

### 1972/4.

Find all solutions  $(x_1, x_2, x_3, x_4, x_5)$  of the system of inequalities

$$\begin{aligned}(x_1^2 - x_3x_5)(x_2^2 - x_3x_5) &\leq 0 \\(x_2^2 - x_4x_1)(x_3^2 - x_4x_1) &\leq 0 \\(x_3^2 - x_5x_2)(x_4^2 - x_5x_2) &\leq 0 \\(x_4^2 - x_1x_3)(x_5^2 - x_1x_3) &\leq 0 \\(x_5^2 - x_2x_4)(x_1^2 - x_2x_4) &\leq 0\end{aligned}$$

where  $x_1, x_2, x_3, x_4, x_5$  are positive real numbers.

### 1972/5.

Let  $f$  and  $g$  be real-valued functions defined for all real values of  $x$  and  $y$ , and satisfying the equation

$$f(x+y) + f(x-y) = 2f(x)g(y)$$

for all  $x, y$ . Prove that if  $f(x)$  is not identically zero, and if  $|f(x)| \leq 1$  for all  $x$ , then  $|g(y)| \leq 1$  for all  $y$ .

### 1972/6.

Given four distinct parallel planes, prove that there exists a regular tetrahedron with a vertex on each plane.