

36th International Mathematical Olympiad

First Day - Toronto - July 19, 1995

Time Limit: 4½ hours

1. Let A, B, C, D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at X and Y . The line XY meets BC at Z . Let P be a point on the line XY other than Z . The line CP intersects the circle with diameter AC at C and M , and the line BP intersects the circle with diameter BD at B and N . Prove that the lines AM, DN, XY are concurrent.
2. Let a, b, c be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

3. Determine all integers $n > 3$ for which there exist n points A_1, \dots, A_n in the plane, no three collinear, and real numbers r_1, \dots, r_n such that for $1 \leq i < j < k \leq n$, the area of $\triangle A_i A_j A_k$ is $r_i + r_j + r_k$.

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Second Day - Toronto - July 20, 1995

Time Limit: 4½ hours

1. Find the maximum value of x_0 for which there exists a sequence $x_0, x_1, \dots, x_{1995}$ of positive reals with $x_0 = x_{1995}$, such that for $i = 1, \dots, 1995$,

$$x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i}.$$

2. Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$ and $DE = EF = FA$, such that $\angle BCD = \angle EFA = \pi/3$. Suppose G and H are points in the interior of the hexagon such that $\angle AGB = \angle DHE = 2\pi/3$. Prove that $AG + GB + GH + DH + HE \geq CF$.
3. Let p be an odd prime number. How many p -element subsets A of $\{1, 2, \dots, 2p\}$ are there, the sum of whose elements is divisible by p ?