## 45rd IMO 2004

**Problem 1.** Let ABC be an acute-angled triangle with  $AB \neq AC$ . The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC. The bisectors of the angles  $\angle BAC$  and  $\angle MON$  intersect at R. Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC.

**Problem 2.** Find all polynomials f with real coefficients such that for all reals a,b,c such that ab+bc+ca=0 we have the following relations

$$f(a-b) + f(b-c) + f(c-a) = 2f(a+b+c).$$

**Problem 3.** Define a "hook" to be a figure made up of six unit squares as shown below in the picture, or any of the figures obtained by applying rotations and reflections to this figure.



Determine all  $m \times n$  rectangles that can be covered without gaps and without overlaps with hooks such that

- the rectangle is covered without gaps and without overlaps
- no part of a hook covers area outside the rectagle.

**Problem 4.** Let  $n \geq 3$  be an integer. Let  $t_1, t_2, ..., t_n$  be positive real numbers such that

$$n^{2} + 1 > (t_{1} + t_{2} + \dots + t_{n}) \left(\frac{1}{t_{1}} + \frac{1}{t_{2}} + \dots + \frac{1}{t_{n}}\right).$$

Show that  $t_i, t_j, t_k$  are side lengths of a triangle for all i, j, k with  $1 \le i < j < k \le n$ .

**Problem 5.** In a convex quadrilateral ABCD the diagonal BD does not bisect the angles ABC and CDA. The point P lies inside ABCD and satisfies

$$\angle PBC = \angle DBA$$
 and  $\angle PDC = \angle BDA$ .

Prove that ABCD is a cyclic quadrilateral if and only if AP = CP.

**Problem 6**. We call a positive integer *alternating* if every two consecutive digits in its decimal representation are of different parity. Find all positive integers n such that n has a multiple which is alternating.